

BLUE PRINT FOR QUESTION PAPER PATTERN
COURSE-V, LINEAR ALGEBRA

Unit	TOPIC	S.A.Q (including choice)	E.Q (including choice)	Marks Allotted
I	Vector spaces - I	2	2	30
II	Vector spaces - II	1	2	25
III	Linear Transformation	2	2	30
IV	Char. values and char. vectors	1	2	25
V	Inner product spaces	2	2	30
Total		8	10	140

S.A.Q. = Short answer questions (5 marks)

E.Q. = Essay questions (10 marks)

Short answer questions : 5 X 5 M = 25 M

Essay questions : 5 X 10 M = 50 M

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Total Marks = 75 M

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CBCS/ SEMESTER SYSTEM
(w.e.f. 2020-21 Admitted Batch)
B.A./B.Sc. MATHEMATICS
COURSE-V, LINEAR ALGEBRA

Time: 3Hrs

Max.Marks:75M

SECTION - A

Answer any FIVE questions. Each question carries FIVE marks 5 X 5 M=25 M

1. Let p, q, r be fixed elements of a field F . Show that the set W of all triads (x, y, z) of elements of F , such that $px+qy+rz=0$ is a vector subspace of $V_3(R)$.
2. Define linearly independent & linearly dependent vectors in a vector space. If α, β, γ are linearly independent vectors of $V(R)$ then show that $\alpha + \beta, \beta + \gamma, \gamma + \alpha$ are also linearly independent.
3. Prove that every set of $(n + 1)$ or more vectors in an n dimensional vector space is linearly dependent.
4. The mapping $T : V_3(R) \rightarrow V_3(R)$ is defined by $T(x, y, z) = (x-y, x-z)$. Show that T is a linear transformation.
5. Let $T: R^3 \rightarrow R^2$ and $H: R^3 \rightarrow R^2$ be defined by $T(x, y, z) = (3x, y+z)$ and $H(x, y, z) = (2x-z, y)$. Compute i) $T+H$ ii) $4T-5H$ iii) TH iv) HT .
6. If the matrix A is non-singular, show that the eigen values of A^{-1} are the reciprocals of the eigen values of A .
7. State and prove parallelogram law in an inner product space $V(F)$.
8. Prove that the set $S = \left\{ \left(\frac{1}{3}, \frac{-2}{3}, \frac{-2}{3} \right), \left(\frac{2}{3}, \frac{-1}{3}, \frac{2}{3} \right), \left(\frac{2}{3}, \frac{2}{3}, \frac{-1}{3} \right) \right\}$ is an orthonormal set in the inner product space $R^3(R)$ with the standard inner product.

SECTION - B

Answer ALL the questions. Each question carries TEN marks. 5 X 10 M = 50 M

- 9(a)) Define vector space. Let $V(F)$ be a vector space. Let W be a non empty sub set of V . Prove that the necessary and sufficient condition for W to be a subspace of V is $a, b \in F$ and $\alpha, \beta \in V \Rightarrow a\alpha + b\beta \in W$.

(OR)

- (b) Prove that the four vectors $(1,0,0)$, $(0,1,0)$, $(0,0,1)$ and $(1,1,1)$ of $V_3(\mathbb{C})$ form linearly dependent set, but any three of them are linearly independent.

- 10(a) Define dimension of a finite dimensional vector space. If W is a subspace of a finite dimensional vector space $V(F)$ then prove that W is finite dimensional and $\dim W \leq n$.

(OR)

- (b) If W be a subspace of a finite dimensional vector space $V(F)$ then Prove that $\dim V/W = \dim V - \dim W$.

- 11(a) Find $T(x, y, z)$ where $T: \mathbb{R}^3 \rightarrow \mathbb{R}$ is defined by $T(1, 1, 1) = 3$, $T(0, 1, -2) = 1$, $T(0, 0, 1) = -2$

(OR)

- (b) State and prove Rank Nullity theorem.

- 12(a) Find the eigen values and the corresponding eigen vectors of the matrix

$$A = \begin{pmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{pmatrix}.$$

(OR)

- (b) State and prove Cayley-Hamilton theorem.

- 13(a) State and prove Schwarz's inequality in an Inner product space $V(F)$.

(OR)

- (b) Given $\{(2,1,3), (1,2,3), (1,1,1)\}$ is a basis of $\mathbb{R}^3(\mathbb{R})$. Construct an orthonormal basis using Gram-Schmidt orthogonalisation process.